Compensating The Dead[*]

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Abstract

An early death is, undoubtedly, a serious disadvantage. However, the compensation of short-lived individuals has remained so far largely unexplored, probably because it appears infeasible. Indeed, short-lived agents can hardly be identified *ex ante*, and cannot be compensated *ex post*. We argue that, despite those difficulties, a compensation can be carried out by encouraging early consumption in the life cycle. In a model with heterogeneous preferences and longevities, we show how a specific social criterion can be derived from intuitive principles, and we study the corresponding optimal policy under various informational assumptions.

Keywords: compensation, longevity, mortality, fairness, redistribution.
JEL codes: D63, D71, I18, J18.

1 Introduction

It is undeniably true that an early death constitutes a serious loss, even when it is due to natural causes. Such a loss should, in a fair society, imply a compensation. However, the compensation of short-lived persons has remained so far largely unexplored in policy circles. The absence of debate on that issue is surprising, since longevity inequalities are widely documented. It is well-known that sizeable longevity differentials exist even within a given cohort, as shown by Figure 1.[1] Although all cohort members are, by definition, born in the same country and at the same epoch, there is a substantial dispersion of the age at death, some persons turning out to have longer lives than others.
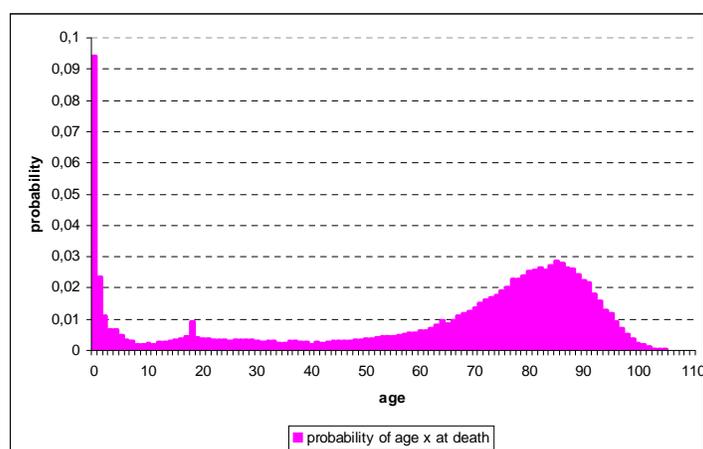

Figure 1: Distribution of the age at death: Swedish female (1900 cohort)

Given that longevity differentials are mainly explained by factors on which individuals have, on their own, little control, there exists a strong ethical intuition for compensating short-lived agents, who are, in some sense, victims of the arbitrariness of Nature.[2] Longevity inequalities due to differences in genetic backgrounds are the best illustration of this. According to Christensen *et al* (2006), about one quarter to one third of longevity inequalities within a cohort can be explained by differences in

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[1]Sources: the Human Mortality Data Base (2009).

[2]Note that longevity is also influenced by individuals, for instance through their lifestyles (see Kaplan *et al* 1987), but those behavioural determinants of longevity (e.g. smoking, diet, physical activity, etc.) only explain one part of longevity differentials, the rest remaining out of individuals' control (e.g. genetic background, environmental determinants of longevity, etc.).

the genetic background. Hence there is a strong intuitive support for compensating the short-lived, who cannot be regarded as responsible for their genes.

But despite the sizeable — and largely arbitrary — longevity differentials, little attention has been paid to the compensation of short-lived agents. The reason behind that lies in the apparent impossibility to compensate short-lived individuals. A first difficulty is that short-lived agents can hardly be identified *ex ante*. Life-tables statistics show the distribution of the age at death in a population or a subpopulation (e.g., by gender), but do not tell us what the longevity of each individual will be.[3] Another difficulty is that, *ex post*, i.e., once a short-lived person is identified, its well-being can no longer be affected, so that little compensation can take place at that stage.[4] Thus we face a non-trivial compensation problem: agents to be compensated cannot be identified *ex ante*, and cannot be compensated *ex post*. Such difficulties may explain why little attention has been paid to the compensation of an early death.

This problem seems in sharp contrast to the opposite problem of helping the elderly people who are in poverty. As they can be identified *ex post* and benefit from income support, it appears straightforward to organize a social security system in their favor.

The goal of the present paper is to propose a way to overcome the difficulties with the compensation of the short-lived. For this purpose, the first part of this paper is devoted to the construction of a measure of social welfare in the context of unequal longevities. The social objective is derived from basic principles guaranteeing that compensating the agents who turn out to be short-lived would be desirable. Moreover, the approach, of the "egalitarian-equivalent" type, takes the agents' preferences over longevity into account.[5] More precisely, the proposed social objective evaluates a particular social state by looking at the smallest consumption the individuals would accept in the replacement of their current situation, if they could benefit from some reference longevity level. In sum, it applies the maximin criterion to what we call the Constant Consumption Profile Equivalent on the Reference Lifetime (CCPERL). Hence we shall refer to the social objective we propose as the Maximin CCPERL.

Once the social objective is defined, it can be used to compute the optimal allocation of resources in various environments. In the second part of the paper, we compute the social optimum in a context in which the social planner knows each individual's preferences and life expectancy, as well as the statistical distribution of

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[3]This is problematic because compensation requires information on individual outcomes.

[4]It is true that, in some cases, a premature death could be anticipated before death occurs. But it is unlikely that a significant compensation could be provided during those hard times.

[5]The egalitarian-equivalent approach to equity was first introduced by Pazner and Schmeidler (1978).

longevities in the population (but not individual longevities). We then also consider the more relevant second-best context, in which the planner knows the distribution of all variables (including longevity), but ignores each individual's preferences and life expectancy. It might seem that very little compensation for a short life can be made in this case, but the planner can nonetheless improve the lot of the short-lived agents by inducing everyone to consume more in the first part of their life, and less in the second part, than they spontaneously would. One of the results of this paper is that it is even possible, in rather general cases, to eliminate welfare inequalities between short-lived and long-lived agents.

Finally, it should be stressed here that the policy recommendations implied by the Maximin CCPERL social objective, although uncommon, are nonetheless far less counterintuitive than the redistributive implications of utilitarianism in the context of unequal longevities. Actually, as shown by Bommier *et al* (2011a, 2011b) and Leroux *et al* (2011), utilitarianism tends, under standard assumptions like time-additive lifetime welfare and expected utility hypothesis, to redistribute resources from short-lived agents towards long-lived agents, against any intuition of compensation.[6] Our approach is, in this light, more intuitive and attractive than utilitarianism.

The rest of the paper is organized as follows. Section 2 introduces the compensation problem, and provides the intuition for the approach and the main results. Section 3 presents the formal framework. Section 4 derives a social objective from ethical axioms. Section 5 characterizes the optimal allocation of resources under Maximin CCPERL in an economy with heterogeneous preferences and life expectancies, under perfect information of agents' *ex ante* characteristics (first-best problem) and asymmetric information of those characteristics (second-best problem). Section 6 explores some variations on the assumptions. Section 7 examines how the optimal policy can be implemented by a savings tax in a growth model with production. Section 8 computes the actual distribution of CCPERL in France (2008), and illustrates the magnitudes of welfare losses due to premature deaths. Section 9 concludes.

2   Compensating the dead: a primer

To illustrate the challenges raised by the compensation of unequal longevities, let us start with a simple example. Consider a one-cohort, two-period model, with a large population of individuals facing uncertain longevity. They all live the young

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[6]This anti-redistributive bias is due to Gossen's First Law, and is robust to various specifications of lifetime welfare. In particular, as shown by Leroux and Ponthiere (2010), representing lifetime welfare as a concave transform of the sum of temporal utilities only mitigates — but does not eradicate — the utilitarian tendency to redistribute resources towards the long-lived.

age (period 1), but the old age (period 2) is reached with a fixed probability π, which is known to everyone. They are all initially identical, with the same survival probability π, the same endowment w of a storable good, and the same preferences. Their preferences satisfy the expected utility hypothesis with logarithmic temporal welfare and no pure time preferences:

$$U = \ln c + \pi \ln d,$$

where c is consumed in the first period, d in the second. The technology makes it possible to transfer the good from period 1 to period 2 at no cost (and zero return). With an annuity market every agent's budget constraint would be $c + \pi d = w$, but, in order to make the example more realistic,[7] we assume that there is no annuity market, so that the savings of those who die prematurely are not redistributed to the same cohort, and the budget constraint in the laissez-faire is then $c + d = w$.

The equilibrium of the laissez-faire is therefore such that everyone chooses

$$c = \frac{w}{1 + \pi}, \quad d = \frac{\pi w}{1 + \pi}.$$

Because surviving is uncertain and there are no returns to savings, d is lower than c.

The population is assumed to be so large that it is a good approximation to consider that, with probability one, there will be a proportion $1 - \pi$ of individuals who will die prematurely. For them, the realized utility at the end of their life, as they see it at time of death, is equal to $\ln c$. Those who live the second period have a final utility of $\ln c + \ln d$. Individuals know *ex ante* that these will be their final utilities in the two possible outcomes for each of them. They also know that the best plan if they happen to die prematurely would be $c = w$, whereas the best plan if they survive would be $c = d = w/2$. The consumption plan they choose under uncertainty involves too much saving for the former case, too little for the latter.

Let us assume that the mechanism of mortality is deterministic, so that it is already determined in the beginning of period 1 who will die prematurely. Unfortunately, this information is hidden until imminent death or survival is announced to every individual at the end of period 1.

The government, however, can rely on the fact that the proportion of survivors is known. It is already known in the beginning of period 1 that there is a proportion $1 - \pi$ of individuals who save too much for their own interests, and that the rest

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[7]This makes the example more realistic for the problem of old-age poverty. As the paper is about the opposite problem of old-age affluence, nothing in our analysis depends on this assumption. In fact, in this simple example, the problem of the disadvantage of a short life would be even more serious in the presence of an annuity market.

save too little. Without knowing the identity of the individuals belonging to either category, the only thing the government can do in the first period is to encourage or discourage savings for all individuals. Encouraging savings will benefit the survivors, discouraging savings will benefit the short-lived. But every intervention will also hurt the other category of individuals.[8]

Deciding whether to intervene should then depend on which type of individual is disadvantaged. Any intervention will then reduce or expand the inequality due to mortality.

Consider the case of a relatively poor economy in which $\ln d < 0$ (i.e., $w < \frac{1+\pi}{\pi}$) so that the long-lived are worse-off than the short-lived. This situation was not uncommon in the 19th century, when the old workers who became unable to work and were not supported by their children filled the poorhouses. A solution to this problem may be to encourage savings. In particular, if $1 + \pi < w < \frac{1+\pi}{\pi}$, it is possible to make utility positive in both periods by introducing compulsory savings and redistributing this tax among the survivors, so that $d > 1$. In short, one can understand social security as conceived precisely to help those who risk being unable to meet their basic needs in old age. Moreover, a pay-as-you-go system mimics an annuity market.

The cost of very old age, nowadays, reproduces a similar situation in which some people may end up unable to meet their basic needs, and this once again triggers debates about whether to further encourage savings.

Preventing misery in old age is a familiar idea. What we want to point out in this paper is that in a relatively rich economy, one has an opposite problem. If $w > \frac{1+\pi}{\pi}$, one has $\ln d > 0$ and the worse off are the short-lived. In this case, the laissez-faire generates inequalities in favor of the long-lived, and such inequalities can be reduced by discouraging savings. Moreover, by reducing d to one, it is even possible to completely eliminate the inequality between the short-lived and the long-lived.

Achieving $d = 1$ (and $c = w - \pi$) is actually the way to eliminate inequalities between short-lived and long-lived for a poor economy as well as a rich economy. One can even modify the model and introduce inequalities of endowment between the individuals. The government is then faced with the problem that for the poor individuals whose endowment is less than $\frac{1+\pi}{\pi}$, the worse off are the long-lived, whereas the better endowed individuals are affected by the opposite inequality. A general policy that encourages savings across all levels of wealth will help the poor survivors

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[8]It is worth noting that in the context of a single cohort of identical individuals, there is perfect symmetry between the problem of compensating and compensating the short-lived. Even though the long-lived are observed in the second period, it is then too late to organize transfers in their favor when they are all identical.

but will hurt the rich who die prematurely, and conversely when saving is discouraged. But, here again, if it is possible to obtain $d = 1$ in all strata of the population, the inequality due to mortality is eliminated.

This simple example raises several questions. First, can the reduction of the inequality due to mortality be justified by a well-founded social welfare function? The utilitarian social welfare function, for instance, does not incorporate aversion to inequality in utility and, in the above example, would not recommend any intervention in absence of inequalities of endowment. Actually, when there are differences in survival probability, it even recommends transferring resources to those with a better life expectancy because of their ability to benefit from resources over two periods, thereby worsening the fate of the short-lived who have a low life expectancy (in a rich economy in which they are worse off than the long lived).

Second, how is the goal of improving social welfare to be implemented when the individuals have heterogenous preferences over the life cycle and unequal life expectancies?

The rest of this paper addresses these two issues, constructing social preferences that make it possible to compare individuals who differ not only in longevity but also in other characteristics, and examining the implications for policy.

A preliminary consideration must, however, be examined. It was assumed in the example that mortality is already determined at the time of allocation decision in period 1. This assumption was introduced in order to make it clear that the actual interests of some of the individuals are better served by government intervention than by the laissez-faire allocation. But nothing would be changed if mortality was truly random, provided that the proportion of premature deaths is known in advance. This proportion is the relevant information.

Whether mortality is deterministic or not, one could still object that, at the time of decision, government intervention is not welcomed by any individual because it reduces expected utility. But what really counts for the individuals is their final utility, which they obtain under full information, not their expected utility which suffers from their initial lack of information. They know that if they are unlucky, they will be grateful for the intervention. Moreover, while there is uncertainty at the individual level, there is no uncertainty for an impartial observer who knows the proportion of short-lived individuals. It would be irrational for a government to let the future unlucky maximize their expected utility and act against their final interests, when such a government knows the proportion of such people, considers that they are substantially worse off than the others, and can induce them to make a better choice for their informed preferences, at a reasonable cost to the rest of the population.

To emphasize the irrationality involved, a government that would cater to individuals' expected utility rather than their final utilities would have the following preferences. It would prefer the laissez-faire allocation to the $d = 1$ allocation when the identity of the short-lived is not known but would have the reverse preferences when the identity of the short-lived is known. The only difference between the two situations is information about the *identity* of the short-lived, their number being known in both cases. As such a government is impartial, additional information about the identity of individuals is irrelevant. This means that irrelevant information would reverse the preferences of such a government.[9]

It is in fact rather common to use statistical knowledge about numbers in policy-making. Seat-belt rules are not meant to enhance the expected utility of the whole population (otherwise a rule would not really be needed), but to improve the distribution of utility for those who will have an accident (it is believed that seat belts harm some of them but save many more, so that the final distribution of utility is improved). Their number is roughly known in advance, and there is a consensus that they are worse off than those who do not have an accident.[10]

3   The general framework

The model describes the situation of a given finite population of agents with ordinal preferences over lifetime consumption profiles. We consider a pure exchange economy, because the central tenets of the compensation problem can be captured in absence of production.

The set of natural integers (resp., real numbers) is denoted $\mathbb{N}$ (resp., $\mathbb{R}$). Let N be the set of individuals, with cardinality $|N|$. The maximum possible lifespan for any individual, i.e., the maximum number of periods that can ever be lived, is denoted by T, with $T \in \mathbb{N}$ and $T > 1$.

Each individual will have a particular *lifetime consumption profile*. Under the assumption of non-negative consumptions, a lifetime consumption profile for an individual $i \in N$ is a vector of dimension T or less, that is, it is an element x_i in the set $X = \bigcup_{\ell=1}^{T} \mathbb{R}_+^\ell$. The longevity of an individual i with consumption profile x_i is defined by a function $\lambda : X \to \mathbb{N}$, such that $\lambda(x_i)$ is the dimension of the lifetime

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[9]There are libertarian views according to which collective irrationality is acceptable, the most important thing being to let individuals do what they want at the time of decision. In this paper we stick to the orthodox, rational approach to welfare economics.

[10]This argument applies only when there is no aggregate uncertainty. The situation is quite different when there is a macroeconomic risk, because it then makes sense to rely on individuals' expected utilities. For an analysis of the evaluation of micro and macro risks, see Fleurbaey (2010).

consumption profile, that is, the length of existence of individual i.

An *allocation* defines a consumption profile for each individual in the population N. Formally, an allocation for N is a vector $x_N := (x_i)_{i \in N} \in X^{|N|}$.

Each individual $i \in N$ has well-defined *preferences* over life-cycle lotteries. As we will be dealing only with situations in which the distribution of final individual situations is known ex ante, we only need preferences about lifetime consumption profiles X, which makes it possible to simplify concepts and notations.[11] Individual i's preferences are therefore described by an ordering R_i (i.e. a reflexive, transitive and complete binary relation) over X. For all $x_i \in X$, the indifference set at x_i for R_i is defined as $I(x_i, R_i) := \{y_i \in X \mid y_i I_i x_i\}$. For any lives x_i and y_i of equal length, preferences over x_i and y_i are assumed to be continuous, convex and weakly monotonic (i.e. $x_i \geq y_i$ implies $x_i R_i y_i$ and $x_i \gg y_i$ implies $x_i P_i y_i$).[12] Moreover, we assume that for all $x_i \in X$, there exists $(c, ..., c) \in \mathbb{R}_+^T$ such that $x_i I_i (c, ..., c)$, which means that no lifetime consumption profile is worse or better than all lifetime consumption profiles with full longevity. This excludes lexicographic preferences for which longevity is an absolute good or bad. Let $\Re$ be the set of all preference orderings on X satisfying these properties. A *preference profile* for N is a list of preference orderings of the members of N, denoted $R_N := (R_i)_{i \in N} \in \Re^{|N|}$.

Figure 2 shows an example of agents' preferences in a two-period setting, i.e. for $x_i \in \mathbb{R}_+ \cup \mathbb{R}_+^2$. An agent who lives the first period only remains on the horizontal axis (i.e. second period consumption is zero). The dashed line segments mean that the individual is indifferent between the two extreme points of the line segment. The upper end of the dashed segment gives the (constant) consumption that should

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[11]Preferences on certain lifetime consumption profiles are included in standard preferences on lotteries of life, since these consist merely of preferences on degenerate lotteries. For instance, individual i's preferences over lotteries might take the standard form

$$\sum_s p_s \sum_{t=1}^{\lambda(x_i^s)} \frac{1}{(1+\delta)^t} u_i (x_{it}^s),$$

where p_s is the probability of state s, $\lambda(x_i^s)$ the longevity in state s, and x_{it}^s is the consumption at time t in state s. Such preferences imply preferences over lifetime consumption profiles x_i represented by

$$\sum_{t=1}^{\lambda(x_i)} \frac{1}{(1+\delta)^t} u_i (x_{it}).$$

[12]Note that we do not assume those properties for lives with *different* lengths. For instance, requiring that the three-periods life $(2, 2, 1)$ be necessarily better than the two-period life $(2, 2)$ would be too strong. One may prefer death to an additional period with consumption equal to 1.

be given to the agent in each period of a hypothetical two-period life to make him exactly as well-off as he is with a single period of life.
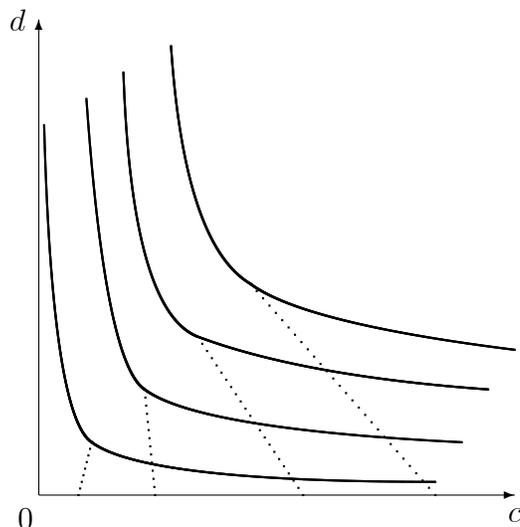

Figure 2: Indifference curves in (c, d) space

Figure 2 illustrates that, to keep the same satisfaction level while raising the length of life, what is required may possibly be either a smaller or a larger consumption per period, depending on the consumption enjoyed while having a short life. For a short-lived individual whose consumption is high (i.e. at the right of the horizontal axis), the consumption that should be given to him in a two-period life to make him indifferent with its current state, which is given by the end of the dashed segment, would be much smaller than its current consumption. This reflects the attractiveness of a longer life for a person with a high current standard of living. On the contrary, for a short-lived agent whose consumption is low (i.e. at the left of the horizontal axis), the consumption that should be given to him in a two-period life to make him indifferent with its current state might be larger than his current consumption. His low current consumption puts him in such a misery that the lengthening of his life with the same consumption per period would make him worse off. Hence additional consumption per period is needed to compensate him for having a longer life.

Clearly, all allocations are not equivalent in terms of how short-lived agents are treated. Therefore, in order to provide theoretical foundations to the compensation of short-lived persons, it is necessary to define *social preferences* over allocations. Such social preferences will serve to compare allocations in terms of their goodness and fairness. Those social preferences will be formalized by a *social ordering function*

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$\succsim$ which associates every admissible preference profile R_N of the population with an ordering $\succsim_{R_N}$ defined on the set of all possible allocations for N, that is, an ordering defined on $X^{|N|}$. For all $x_N, y_N \in X^{|N|}$, $x_N \succsim_{R_N} y_N$ means that the allocation x_N is, under the preference profile R_N, at least as good as the allocation y_N. The symbols $\succ_{R_N}$ and $\sim_{R_N}$ will denote strict preference and indifference, respectively.

Note that, because the actual longevity of each individual is not known and only the distribution is known, what a policy-maker considers is never an allocation x_N, but the set of allocations that have the same distribution of bundles and preferences as (x_N, R_N). Designing preferences over such sets is, however, equivalent to designing preferences over allocations x_N for an impartial policy-maker. This is why we can work with the simpler framework of allocations.

4   The social objective

This section aims at deriving a social objective that is adequate for the allocation of resources among agents having unequal longevities. As mentioned above, standard social objectives like utilitarianism do not do justice to basic intuitions supporting the compensation of the short-lived, so that we need to look for other objectives. Obviously, there exist many possible social preferences. The only way to select reasonable social preferences is to impose some plausible ethical requirements that these should satisfy. Such ethical requirements will take here the form of four axioms.

The first axiom states that if all individuals prefer one allocation to another, then this should also be regarded as socially preferable to that alternative.

Axiom 1 (Weak Pareto) *For all preference profiles $R_N \in \Re^{|N|}$, all allocations $x_N, y_N \in X^{|N|}$, if $x_i P_i y_i$ for all $i \in N$, then $x_N \succ_{R_N} y_N$.*

That axiom can be justified on two grounds. First, it seems essential to respect individual preferences in order to address trade-offs between, for instance, consumption at different points in life. Second, the Pareto axiom is also a guarantee against the choice of inefficient allocations: it states that any unanimity in the population regarding the ranking of two allocations should be respected by social preferences.

The next axiom requires social preferences to use the relevant kind of information about individual preferences. More precisely, it states that, in order to compare allocations, it is sufficient to look at the indifference sets of the individuals at the consumption profiles under consideration.

Axiom 2 (Hansson Independence) *For all preferences profiles $R_N, R'_N \in \Re^{|N|}$ and for all allocations $x_N, y_N \in X^{|N|}$, if for all $i \in N$, $I(x_i, R_i) = I(x_i, R'_i)$ and $I(y_i, R_i) = I(y_i, R'_i)$, then $x_N \succsim_{R_N} y_N$ if and only if $x_N \succsim_{R'_N} y_N$.*

This condition, which was first introduced by Hansson (1973) and Pazner (1979), requires that social preferences over two allocations depend only on the individual indifference curves at these allocations. Note, however, that those indifference curves contain more information than individual pairwise preferences over these two allocations. This allows us to avoid Arrow's impossibility result.

The next two axioms are refinements of the Pigou-Dalton principle in the context of unequal longevities.

The Pigou-Dalton principle for Equal Preferences and Equal Lifetimes is an immediate translation of the Pigou-Dalton principle in the present context. It states that, if we take two allocations such that the consumption profiles are exactly the same under the two allocations for everyone *except* for two persons, then, if those two individuals have equal lifetimes and equal preferences, the allocation in which the two agents have, when alive, closer consumption profiles is more socially desirable than the one in which they have more unequal consumption profiles.

Axiom 3 (Pigou-Dalton for Equal Preferences and Equal Lifetimes) *For all $R_N \in \Re^{|N|}$, all x_N, $y_N \in X^{|N|}$, and all i, $j \in N$, if $R_i = R_j$ and if $\lambda(x_i) = \lambda(y_i) = \lambda(x_j) = \lambda(y_j) = \ell$, and if there exists $\delta \in \mathbb{R}^\ell_{++}$ such that*

$$y_i \gg x_i = y_i - \delta \gg x_j = y_j + \delta \gg y_j$$

and $x_k = y_k$ for all $k \neq i, j$, then

$$x_N \succsim_{R_N} y_N.$$

That axiom is pretty intuitive: for agents who are identical in terms of everything (i.e. longevities, preferences) *except* their consumptions, a redistribution (without any loss) from the agent with the higher consumption to the agent with the lower consumption constitutes a social improvement.

While that refined version of the Pigou-Dalton principle is intuitive, it is nonetheless restricted to agents with equal preferences, which is a strong restriction. Actually, we would like also to be able to say whether a consumption transfer is a social improvement or not when agents have *different* preferences. Note, however, that making this kind of statement is not trivial, as it is not obvious to see in which case some consumption transfer from a rich to a poor could be regarded as a social improvement whatever individual preferences are.

In the following axiom, it is argued that, if the two agents in question have a longevity that is equal to a level of reference ℓ^*, then a transfer that lowers the constant consumption profile of the rich and raises the constant consumption profile of the poor constitutes a social improvement, whatever individual preferences are.

Axiom 4 (Pigou-Dalton for Constant Consumption and Reference Lifetime)
For all $R_N \in \Re^{|N|}$, all x_N, $y_N \in X^{|N|}$, and all i, $j \in N$, such that $\lambda(x_i) = \lambda(y_i) = \lambda(x_j) = \lambda(y_j) = \ell^*$, and x_i and x_j are constant consumption profiles, if there exists $\delta \in \mathbb{R}_{++}^{\ell^*}$ such that

$$y_i \gg x_i = y_i - \delta \gg x_j = y_j + \delta \gg y_j$$

and $x_k = y_k$ for all $k \neq i, j$, then

$$x_N \succsim_{R_N} y_N.$$

The reference longevity level ℓ^* can be interpreted in the following way. An external observer could, when comparing the lives of two persons with the same length ℓ^*, say who is better off than the other by just looking at the constant consumption profiles of those agents, *without* knowing anything about their preferences. Thus, ℓ^* is the length of life such that if it is enjoyed by distinct persons, one can compare the well-being of those agents directly from their consumptions (provided they are constant over time), without caring for their preferences. Note that this axiom is weak. It would be tempting to extend it to cases in which the longevity of the agents can take other values than a particular ℓ^* : isn't it intuitive that a greater constant consumption for any given longevity makes one better off? Unfortunately, such an extension would render the axiom incompatible with Weak Pareto.[13] This is why the axiom can be formulated for at most one reference level of longevity.

There is no need, at this stage, to assign a specific level to the reference longevity ℓ^*. Intuitively, it makes sense to set ℓ^* at the "normal" lifespan, that is, the lifespan that everyone — whatever one's life-plans are — would like to have, but it is not trivial to see which lifespan is the normal one. Note that the selection of ℓ^* may have important redistributive consequences, in combination with the Pareto axiom. Taking, for instance, a maximal lifespan of 120 years as the reference would imply giving priority to those who have a strong preference for longevity. This is because their situation is equivalent, according to their own preferences, to a situation in which they live for 120 years with a low consumption. Given that the "normal" lifespan may vary with the circumstances — in particular with the quality of life (health status) —, we will not fix it, and keep it as an ethical parameter.[14]

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[13]Such an incompatibility between the Pareto principle and the principle of transfer in the multidimensional context is well documented. See, for instance, Fleurbaey and Maniquet (2011). Intuitively, the problem stems from the fact that, at a low common level of longevity, making a *progressive* transfer from an individual who cares a lot about longevity to another who cares less about longevity may be Pareto equivalent to a *regressive* transfer at a larger level of longevity — their indifference curves crossing at an intermediate level of longevity.

[14]It is indeed likely that societies with a better health will consider that ℓ^* is larger.

The four ethical principles that are presented above seem quite reasonable. We now have to investigate which kind of social preferences do satisfy these conditions. As we shall see, the answer to that question will be quite precise. But before providing that answer, let us first introduce what we shall call the Constant Consumption Profile Equivalent on the Reference Lifetime (CCPERL).

Definition 1 (Constant Consumption Profile Equivalent on the Reference Lifetime)
For any $i \in N$, any $R_i \in \Re$ and any $x_i \in X$, the Constant Consumption Profile Equivalent on the Reference Lifetime (CCPERL) of x_i is the constant consumption profile $\hat{x}_i$ such that $\lambda(\hat{x}_i) = \ell^*$ and

$$x_i I_i \hat{x}_i.$$

The CCPERL can be interpreted as a way to *homogenize* consumptions across individuals having different longevities, by converting consumptions under different longevities into some comparable consumptions. The intuition behind that homogenization exercise is the following. In the present context, where agents have unequal longevities, looking at individual consumption profiles does not suffice to have a precise idea of individual well-being. However, the CCPERL does allow to have a more precise view, as it has, by construction, taken longevity differentials into account.

It is trivial to see that, if x_i is a constant consumption profile with $\lambda(x_i) = \ell^*$, then $\hat{x}_i = x_i$. However, if x_i is a constant consumption profile (with consumption level for each life-period equal to c_i) with $\ell < \ell^*$, then we have $\hat{x}_i \gtreqless (c_i, ..., c_i)$, depending on whether c_i lies above or below the critical level making a longer life with that consumption worth being lived. The CCPERL of x_i always exists if $\ell^* = T$, by assumption made on $\Re$, but the existence of the CCPERL is not guaranteed if $\ell^* < T$. It may happen that x_i with high longevity is strictly preferred to all lifetime consumption profiles with lower longevity ℓ^*. When this happens, we adopt the convention that the CCPERL is infinite. This problem of non-existence is not very important as the social preferences highlighted here focus on the worst-off individuals.

Having defined the CCPERL, we can now present the following proposition, which characterizes the social preferences, or, more precisely, states that the Maximin on CCPERL is a necessary condition for social optimality.

Proposition 1 *Assume that the social ordering function $\succsim$ satisfies Axioms 1-2-3-4 on $\Re^{|N|}$. Then $\succsim$ is such that for all $R_N \in \Re^{|N|}$, all x_N, $y_N \in X^{|N|}$,*

$$\min_{i \in N}(\hat{x}_i) > \min_{i \in N}(\hat{y}_i) \implies x_N \succ_{R_N} y_N.$$

In other words, the social ordering satisfies the Maximin property on the Constant Consumption Profile Equivalent on the Reference Lifetime (CCPERL).

The proof is in the Appendix. It should be noted that this proposition does not give a full characterization of social preferences because it does not say how to compare allocations for which $\min(\hat{x}_i) = \min(\hat{y}_i)$.[15] All the proposition states is that if one allocation exhibits a higher minimum CCPERL than another, then it must also be socially more desirable. In other words, the proposition implies that maximizing $\min(\hat{x}_i)$ is a *necessary* operation, as the best social allocation is necessarily included in the set of allocations that maximize $\min(\hat{x}_i)$.

Proposition 1, although being a partial characterization result, tells us a lot about social preferences. True, if the set of allocations that maximize $\min(\hat{x}_i)$ is not a singleton, looking at the minimum CCPERL only would not tell us which allocation is the most desirable. However, in more concrete problems, it is likely that the Maximin on CCPERL has, as a solution, a *unique* allocation, in which case that allocation must also be the most socially desirable allocation. When a unique solution is not obtained, it is natural to refine the Maximin into the Leximin, which extends the lexicographic priority of the worse-off to higher ranks in the distribution.

While the details of the proof are provided in the Appendix, its overall form can be briefly given here. The proof proceeds in two stages. In a first stage, it is shown that Weak Pareto, Hansson Independence and Pigou-Dalton for Equal Preferences and Equal Lifetimes imply Hammond Equity for Equal Preferences. That principle states that, if two persons i and j have the same preferences, but i lies on a higher indifference curve than j, pushing i on a lower indifference curve and j on a higher one is socially desirable. This embodies an absolute priority for the worst-off. In a second stage, Hammond Equity for Equal Preferences is then used to show that, if we add Pigou-Dalton for Constant Consumption and Reference Lifetime, we obtain Hammond Equity for Reference Lifetime. According to that principle, if two persons i and j, possibly with different preferences, have the same longevity equal to the reference ℓ^*, but i has a higher constant consumption profile than j, then lowering the constant consumption profile of i and raising the one of j is socially desirable.

Let us note that an alternative characterization can be made in a slightly different setting. Suppose for the rest of this section that longevity is a continuous variable, so that a lifetime consumption profile is now described as a function $x_i(t)$ defined over the interval $[0, T]$. We restrict attention to functions $x_i(t)$ which are strictly positive and continuous over an interval $[0, \lambda(x_i)]$ and null over the complement $(\lambda(x_i), T]$. The corresponding longevity is obviously $\lambda(x_i)$. Individual preferences over lifetime

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[15]Clearly, given the postulated axioms, the equality of the $\min(\hat{x}_i)$ for two allocations does not necessarily imply social indifference between these allocations: an allocation could still be regarded as better than the other (on the grounds of other aspects of the distribution), and the theorem has nothing to say about that.

consumption profiles x_i can still be defined and assumed to be convex, continuous (with respect to the topology of pointwise convergence) and weakly monotonic. The axioms of Weak Pareto and Hansson Independence are immediately adapted to this setting. Let us now introduce a new axiom which states that, whatever the individual preferences, it is always socially desirable to reduce longevity inequalities among agents who enjoy the same consumption per life-period, when one agent lives longer than ℓ^* and the other has a shorter life. For this axiom not to be idle, it must be assumed that $0 < \ell^* < T$. A similar assumption was not needed with the axiom of Pigou-Dalton for Constant Consumption and Reference Lifetime.

Axiom 5 (Inequality Reduction around Reference Lifetime) *For all $R_N \in \Re^{|N|}$, all x_N, $y_N \in X^{|N|}$, and all i, $j \in N$, such that $\lambda(x_i) = \ell_i$, $\lambda(y_i) = \ell'_i$, $\lambda(x_j) = \ell_j$ and $\lambda(y_j) = \ell'_j$, and some $c \in \mathbb{R}_{++}$ is the same constant per-period level of consumption for x_i, y_i, x_j, y_j, if*

$$\ell_j, \ell'_j \leq \ell^* \leq \ell_i, \ell'_i \ and \ \ell_j - \ell'_j = \ell'_i - \ell_i > 0$$

and $x_k = y_k$ for all $k \neq i, j$, then

$$x_N \succsim_{R_N} y_N.$$

That axiom is quite attractive: reducing longevity inequalities between long-lived and short-lived agents who enjoy equal consumptions per period can hardly be regarded as undesirable. Note, however, that the attractiveness of that axiom is not independent from the monotonicity of preferences in longevity. If consumption per period is so low that some agents may prefer having a short rather than a long life, reducing longevity inequalities by raising the longevity of the short-lived may be socially undesirable. Thus this axiom, unlike axioms 3 and 4, must be used in a subdomain of preferences satisfying a stronger monotonicity condition with respect to longevity.

Observe that by weak monotonicity, for every individual preference ordering R_i and every lifetime consumption profile x_i, there is a unique constant profile with same longevity such that every profile with greater consumption and same longevity is strictly preferred and every profile with lower consumption and same longevity is strictly worse. Therefore, by Weak Pareto one can restrict attention to constant lifetime consumption profiles and work with bundles having two dimensions, namely, per-period consumption and longevity. Formally, Inequality Reduction around Reference Lifetime is then similar to the Free Lunch Aversion Condition proposed by Maniquet and Sprumont (2004) in the context of public goods provision. It is then a

simple adaptation of their analysis to show that the conclusion of Proposition 1 holds in this particular setting when it is required that the social ordering function must obey the axioms 1-2-5. The only minor difference is that longevity is here bounded between 0 and T, whereas the corresponding variable (contribution of private good to the production of public good) is unbounded in their model.[16]

5   Policy evaluation

The previous section showed that basic axioms on social preferences imply that the optimal allocation must maximize the minimum CCPERL in the population. What are the consequences of this result for policy evaluation and for the optimum allocation of resources? If, for instance, a social planner could have anticipated, in 1900, the distribution of longevities of Swedish women as shown on Figure 1, how should he have influenced the distribution of resources among the cohort members?

This section aims at deriving a method for the evaluation of policies in a resource allocation problem when the axioms of Proposition 1 are satisfied. We will also examine, in the light of the optimum allocation of resources, to what extent the Maximin on CCPERL may lead to a partial or a total compensation of short-lived agents.

5.1   Environment

Unlike the previous section that could focus on allocations, we now need a richer description of the economy in order to take account of individual behavior and incentive effects. We will, however, introduce one simplification, which is to consider the case in which the individuals live either one or two periods (i.e. $T = 2$).[17] The first period can be interpreted as the young age of life, which is lived by everyone. The second period, i.e. the old age of life, is enjoyed by some individuals only. The length of life of each agent is only known *ex post. Ex ante*, the social planner knows

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[16]Although Inequality Reduction around Reference Lifetime is meaningful in the model of this paper introduced in Section 3, Th. 1 does not hold with axioms 1-2-5, even with stronger monotonicity assumptions about individual preferences. The reason is that if the worst-off gains very little, this may not be equivalent to gaining one period of longevity at any level of consumption. Axiom 5 is then powerless because it applies only when the worst-off in the "transfer" of longevity gains at least one period of additional longevity.

[17]The assumption $T = 2$ is here made for analytical simplicity (i.e. it reduces the number of groups *ex post*). We discuss below how robust our results are to assuming $T > 2$.

individual preferences and life expectancies, as well as the statistical distribution of longevity in the population.

Individual i is now endowed with preferences over lotteries of the form $(c_i, d_i; \pi_i)$, where c_i is consumption in young age, d_i is consumption in old age, and π_i is the probability of surviving to old age. No uncertainty about consumption is considered here. We assume that preferences follow the expected utility criterion, and can be represented by the formula

$$u_i(c_i) + \pi_i v_i(d_i),$$

with the difference between u_i and v_i embodying various phenomena such as time preference,[18] declining health in old age, and changing abilities or goals in the use of resources. The functions u_i, v_i are continuous and increasing.

If the individual has a short life, his final utility is $u_i(c_i)$. If he has a long life, his final utility is $u_i(c_i) + v_i(d_i)$.

An important role is played in the analysis by the sign of $v_i(0)$. Unless otherwise specified, it is assumed throughout the paper that $u_i(0), v_i(0) < 0$, which means that life is unpleasant in absence of resources. Empirical papers focusing on differential mortality generally make this assumption (see, for instance, Becker *et al.* 2005). It is also assumed that utility becomes positive when consumption is high enough (i.e. beyond a subsistence level).

Under these assumptions, for all (c, d), there are always x, y such that

$$
\begin{aligned}
u_i(c) &= u_i(x) + v_i(x), \\
u_i(c) + v_i(d) &= u_i(y) + v_i(y),
\end{aligned}
$$

as it was assumed in the previous section. Moreover, it is clear that $x \leq y$ if and only if $v_i(d) \geq 0$.

We will assume in this section that the reference longevity for the CCPERL is $\ell^* = 2$. We will examine later how another reference affects the analysis.

It is always assumed in this section that the government observes (c_i, d_i) and can therefore perform redistribution between agents with unequal endowments. This enables us to focus on the problem of compensation between short-lived and long-lived in the presence of heterogeneities of preferences and life expectancies. Informational constraints on the redistribution of wealth are introduced in the next section.

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[18]Time preference as understood here does not represent a myopic bias, but rational preferences about when to consume. Some people may prefer to consume early, other may prefer to compensate old age by comfort.

5.2 First-best optimum

In the first-best context, the social planner knows individual characteristics (u_i, v_i, π_i) but does not know the individuals' longevity *ex ante*. Consistently with the assumption that the planner knows the distribution of final situations, the population is assumed to be large, and in each group of individuals sharing the same u_i, v_i, π_i, it is assumed that it is a good approximation to consider that a proportion π_i will have a long life.

There are conditions under which equalizing the CCPERL across all individuals is impossible. If resources are so scarce that life is unpleasant for everyone at all ages, the short-lived are the lucky ones and the problem is not interesting.

Supposing that resources are sufficient, it is interesting to see if, knowing the *ex ante* characteristics of each individual, the social planner may be able to equalize the CCPERL across all individuals.[19] What does such an allocation look like?

The CCPERL of the short-lived agents is the solution x to the equation

$$u_i(c_i) = u_i(x) + v_i(x),$$

whereas for the long-lived agents it is the solution y to the equation

$$u_i(c_i) + v_i(d_i) = u_i(y) + v_i(y).$$

Achieving equality across long-lived and short-lived individuals is possible by planning a second-period consumption d_i such that $v_i(d_i) = 0$ for all individuals. As suggested in the example of Section 2, making a longer life neutral for the individuals preferences eliminates the advantage or disadvantage of a premature death.

Achieving equality across all individuals also requires choosing the first-period consumption c_i so that the solution x to the equation $u_i(c_i) = u_i(x) + v_i(x)$ is the same for all agents. This implies that the agents who derive a greater utility from the second period, at every level of consumption, will receive more consumption in the first period.

Interestingly, no difference of lifetime consumption plan is observed between agents who have the same u_i, v_i and differ only in π_i. The mere fact of having a greater life expectancy is not viewed as an advantage once uncertainty is resolved. This is because the individuals' final utility, which is either $u_i(c_i)$ or $u_i(c_i) + v_i(d_i)$, does not record the anxiety of a premature death or the hope of living longer.

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[19] There may exist technical constraints to redistribution, for instance if the agents live in different times. We will focus here on the case in which the planner is not prevented by such constraints to equalize the CCPERL across all agents.

Recording such feelings would require a different form of utility in which the utility of the first period would contain a term that represents the expectation for the second period. For instance, if the first-period utility was $u_i(c_i) + \alpha_i \pi_i v_i(d_i)$, where α_i is the parameter capturing the importance of expectations in the individual's appreciation of the first period, then a greater life expectancy would be recorded as an advantage if $v_i(d_i) > 0$ and a disadvantage if $v_i(d_i) < 0$. However, even with this additional term, equalizing final utilities between short-lived and long-lived agents would still require setting $v_i(d_i) = 0$, which would make the feeling factor in the first period disappear as well. Therefore, even under such refinement of the model, a first-best allocation that equalizes the CCPERL across agents would not be sensitive to differential life expectancies.

Note that if for some agents $v_i(0) > 0$, then it is impossible to equalize the CCPERL because, even with $d_i = 0$, the long-lived have an advantage over the short-lived.

The following proposition sums up the results obtained in the first-best problem.

Proposition 2 *Assume that the social planner manages to equalize the CCPERL across all individuals, under perfect information about ex-ante types (u_i, v_i, π_i):*

- *Full compensation for a short life (or a long life) is achieved by having $v_i(d_i) = 0$ for all individuals.*

- *The agents having a greater v_i function, other things equal, have a greater first-period consumption.*

- *Agents who differ in their survival probabilities but have the same preferences are treated identically, even when expectations affect the utility of the first period.*

- *If $v_i(0) > 0$ for some agents, the short-lived among them cannot be fully compensated.*

5.3   Second-best optimum

In the first-best context, individual characteristics u_i, v_i and π_i are perfectly observable by the social planner. This subsection reexamines the egalitarian-equivalent solution under asymmetric information, that is, when agents know their (u_i, v_i, π_i)-type, while the government only observes the distributions of types. The government can still propose different bundles (c, d) to *ex ante* groups, but under the constraint of incentive compatibility.

The set of bundles offered must not contain two bundles $(c, d) > (c', d')$, otherwise no one would choose the latter. Therefore the menu consists of a decreasing budget curve in the (c, d) space.

Let d_i^* denote the level of consumption that gives zero utility in the second period: $v_i(d_i^*) = 0$. Figure 3 provides an illustration of how such d_i^* is connected to the determination of indifference between long life and short life at different levels of consumption.
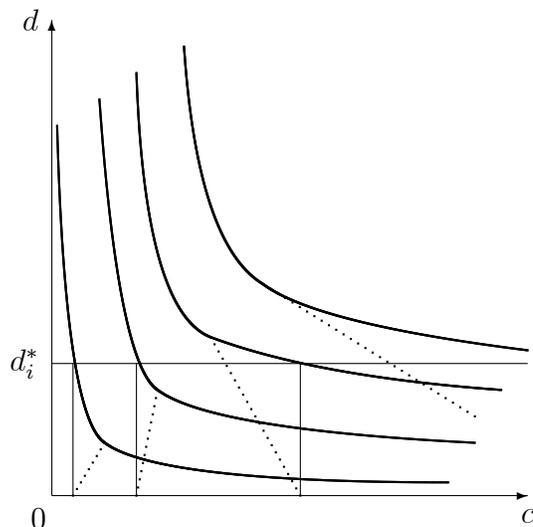

Figure 3: Indifference curves in (c, d) space

If d_i^* is the same d^* for all agents, the planner can still equalize the CCPERL between short-lived and long-lived agents by planning $d_i = d^*$ for all individuals. However, incentive-compatibility then requires equalizing c_i across all individuals. This makes it possible for inequalities of CCPERL between agents with different preferences to remain. The agents with a greater v_i function above d^* will have a lower CCPERL.

This is not necessarily the optimum allocation. It may be possible that, offering another bundle (c', d') such that c' is greater and $d' < d^*$, some agents will choose this alternative option. If they die young, they are better off than those consuming the other bundle. If they survive, they may have a greater CCPERL than the lowest because their time preference, which has induced them to choose this alternative bundle, may imply that they are indifferent between their lifetime plan with a great first period consumption and a lifetime plan with a constant consumption at a reasonably high level.

21

What is the point of offering this additional bundle? If d' is low enough, it may make it possible to free resources in the second period and use more resources in the second period to increase the first-period consumption of the worst off. For this to happen, one needs a low interest rate and the presence of individuals with a strong time preference, which may not be very realistic. An unpalatable feature of this new option is that it makes the long-lived have a negative utility in the second period.

Now let us consider the case of differing values of d_i^*. To make things simpler, we will assume that the agents with a greater d_i^* have a lower function v_i at all values of d.

Suppose that the planner gives all agents the lowest value of d_i^* as the planned consumption for the second period, and the same consumption c in the first period. This nullifies inequalities of CCPERL between agents who have the lowest d_i^* and the same u_i.

Among the short-lived, those who have a greater d_i^* than the lowest obtain a greater CCPERL, because the equation $u_i(c_i) = u_i(x) + v_i(x)$ is solved with a greater x when the function v_i is lowered.

Among the long-lived, the influence of a different v_i is ambiguous. When v_i is lowered (corresponding to a greater d_i^*), the equation $u_i(c_i) + v_i(d_i) = u_i(y) + v_i(y)$ endures a reduction on both sides. If this reduction is the same decrement, the equation is satisfied for the same y, which means that the CCPERL is the same across long-lived agents with different d_i^*.

Therefore, although this depends on the heterogeneity between the v_i functions across agents, it appears that providing the same plan $(c, \min_i d_i^*)$ to all individuals appears a better option than providing a plan involving a greater d_i^*. It does not eliminate inequalities in CCPERL fully, the main inequalities that remain being at the advantage of the short-lived who have a greater d_i^*. But if the variations in the v_i functions are not too different from incremental differences, the long-lived have similar values of the CCPERL as the most disadvantaged short-lived.

With this plan, the long-lived with a greater d_i^* than the lowest obtain a negative utility in old age. Although this is not reflected in a lower CCPERL for them, it is not, once again, an appealing feature of the solution. But if a greater d_i^* was chosen, the short-lived with a low d_i^* would have a lower CCPERL than the other agents and a lower CCPERL than in the contemplated plan. This issue may not deserve much focus because one may consider that differences in d_i^* are small across individuals in a given country. As we will see, this is also something that is sensitive to the choice of the reference longevity.

The following proposition sums up the results of the second-best.

Proposition 3 *Under asymmetric information about ex ante types:*

- *When d_i^* is the same d^* for all i, full compensation for a short (or long) life is achieved when second-period consumptions are all equal to d^*. Incentive compatibility then requires equalizing first-period consumptions for all agents. This typically leaves inequalities to the detriment of agents with a greater v_i function above d^*. This may not be the optimal allocation if offering another option with greater c and lower d may attract individuals with strong time preference and free resources for first-period consumption.*

- *When d_i^* differs across individuals, assuming that differences in v_i are only incremental implies that the best plan involving a single bundle offers $(c, \min_i d_i^*)$, for some c, to all individuals, leaving inequalities to the advantage of short-lived agents with a greater d_i^*.*

- *In both cases, the optimal plan may involve giving a negative utility to some long-lived agents.*

A common feature of the first-best and the second-best optima is that consumptions are, in each case, not smoothed across time when it is possible to give a consumption in the first period that is much greater than d_i^*. Indeed, the compensation of short-lived agents then requires resources to be concentrated at the young age (since this is enjoyed by *all* individuals, either short-lived or long-lived). However, the second-best optimum, when it consists of a single bundle, differs from the first-best by the equal treatment of all agents in the first period of life, whatever their preferences.

The possibility of giving a negative utility to some long-lived agents appears unappealing. It is hard, however, to assess the extent and frequency of this phenomenon. In both cases (uniform d_i^* or non-uniform d_i^*), there are reasons to believe this to be rare.

5.4 The reform problem

In this subsection we examine how the government can evaluate any arbitrary budget curve and contemplate a reform on this curve, on the basis of the Maximin CCPERL. This may be especially relevant if various political or even ethical constraints make it impossible to completely compensate the loss due to a premature death or to redistribute between individuals with unequal wealth. In such a context, one may nevertheless compare the impact of the feasible options on the worst off as identified by the CCPERL index.

Consider first the case in which all individuals face the same budget curve. Assume that in the current budget all individuals choose bundles such that $c_i > d_i^*$. An

individual who chooses a lower c and greater d (than someone else) is typically an individual who has a lower preference for consuming when young and has a greater life expectancy. When such an individual has a short life, her CCPERL is the solution x to the equation $u_i(c_i) + v_i(d_i^*) = u_i(x) + v_i(x)$, with a relatively low c_i and, due to a lower time preference, a relatively low x for a given $c_i > d_i^*$, as illustrated on Figure 4, in which i has a lower time preference than j. Therefore, the individuals with the lowest CCPERL are most likely to be those who choose the lowest consumption in the common budget curve that is offered to individuals.
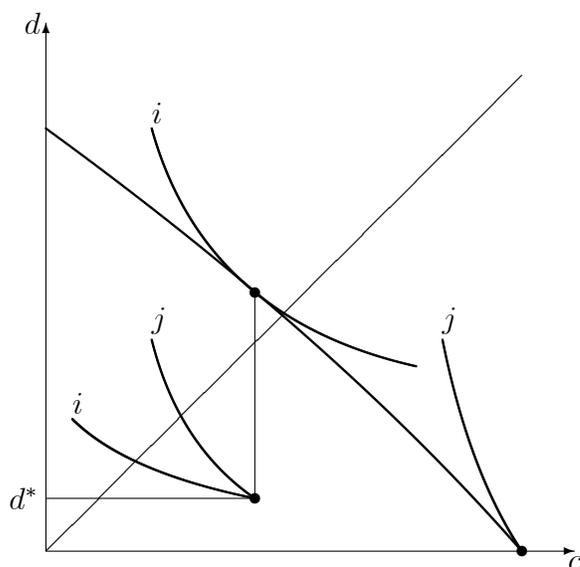

Figure 4: Finding the lowest CCPERL.

In particular, if one assumes that at every level of chosen consumption one finds the same span of values for d_i^*, and that preferences induce single-crossing indifference curves, then one is sure that the lowest CCPERL is to be found among the agents with the lowest c_i. This is proved as follows. The agents who choose the lowest c_i have indifference curves that display a lower time preference than the agents choosing a greater c_i, the expression "lower time preference" being unambiguous thanks to single-crossing. As the same values of d_i^* can be found for all levels of chosen c_i, single-crossing guarantees that at any given level of d_i^*, the lowest CCPERL is found among the agents with lowest time preference, who happen to be those who choose

the lowest c_i.

When individuals have different endowments that induce different budget curves, the worst-off are then the individuals who have the lowest wealth and save the most (keeping the assumption that poverty is never so dire as to induce $c_i \leq d_i^*$). Redistributing wealth and discouraging savings are then two means of increasing the lowest CCPERL.

These considerations deliver the following result.

Proposition 4 *Assuming single-crossing indifference curves and non-crossing budget curves, and $c_i > d_i^*$ for all i :*

- *At a given level of d_i^*, the lowest CCPERL is obtained by the agents with the lowest budget curve and the lowest time preference. These are those who choose the lowest c_i.*

- *Assuming that $\min_i d_i^*$ is present among the agents with the lowest time preference and the lowest budget curve, the reform improves the lowest CCPERL if it raises the lowest c_i (provided that d_i remains above d_i^* for all i).*

6 Variations

In this section we briefly examine how the results are affected when the reference longevity is changed and when the length of life can be greater than two periods.

6.1 Reference longevity

Until now, we have assumed that the reference longevity was the maximum length of life, i.e. $\ell^* = 2$, and computed the CCPERL for all individuals on the basis of that reference longevity. Let us now assume, alternatively, that the reference longevity ℓ^* is the minimum longevity (i.e. 1 period). Under this assumption, the computation of the CCPERL is made by seeking the solutions x, y to the equations

$$
\begin{aligned}
u_i(c_i) &= u_i(x), \\
u_i(c_i) + v_i(d_i) &= u_i(y) + v_i(d_i^*).
\end{aligned}
$$

In the first-best context, equalizing the CCPERL across individuals still requires having $d_i = d_i^*$ for all i. The new result is that now c_i will be equalized across individuals, unlike the previous case in which the agents with a greater v_i function enjoyed a greater c_i.

In the second-best context, when d_i^* is the same d^* for all i, the qualitative results are not affected. Compensation for different longevity is still achieved by offering the same bundle (c, d^*) to all individuals. Offering another bundle with greater $c' > c$ and lower $d' < d^*$, that would attract the individuals with a strong time preference, may still be optimal if this makes it possible to raise c for the worst off. This is slightly less likely than previously, however. Now, this additional option must be such that the long-lived who chose it must prefer their bundle to the single bundle (c, d^*) even after they know that they live two periods, whereas previously it was sufficient that the intersection of their indifference with the 45° line be higher than the lowest CCPERL. This is because the new CCPERL (with $\ell^* = 1$) is then found at the intersection of their indifference curve and the horizontal line $d = d^*$.

When d_i^* differs across individuals, the qualitative results are now reversed. When the same bundle $(c, \min_i d_i^*)$ is offered to all, the worst off are the long-lived agents with a greater d_i^*, and their CCPERL is less than c. It is therefore better to seek to maximize c while offering the same bundle $(c, \max_i d_i^*)$ to all. In this case the worst off are the short-lived and also the long-lived with the greatest d_i^*, and their CCPERL is equal to c, while the long-lived with a lower d_i^* have a greater CCPERL because they are indifferent between $(c, \max_i d_i^*)$ and (c', d_i^*) for a greater c'.

All in all, one should not exaggerate the influence of the reference longevity on the Maximin CCPERL solution. Whatever ℓ^*, it keeps the property of decreasing optimal consumption profiles, in such a way as to compensate short-lived agents.[20]

6.2 The maximum lifespan

So far our study of social evaluation with the Maximin CCPERL was carried out within a model where agents could live either one or two periods. While the focus on the case where $T = 2$ is convenient, one may wonder what our results become under a more realistic demography, where the lifespan includes more periods than a "young age" and an "old age". To examine that issue, we will consider here an economy where $T > 2$, and where there exists a risk of death at each period. Note that if the rise in T consisted merely of adding periods of *certain* life at the beginning of the life-cycle, such a change would be equivalent to an extension of the first period in the basic model, and this would not raise the number of individual situations *ex post*, and could thus hardly affect our results significantly.

We will assume that the "neutral" level of consumption that nullifies utility is the same at all periods. While this may not be very realistic (subsistence needs may

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[20]Similarly, one could explore variants to the reference to a constant consumption in the definition of CCPERL, without finding much change to the main policy conclusions.

vary with age), it is unlikely that this level should greatly vary, and the assumption that it is constant should be understood as an approximation.

Under this assumption, it turns out that full compensation for differential longevity is obtained when consumption is equal to the neutral level at all ages that an agent is not sure to reach. Only the ages for which existence is certain can have a greater level.

This being noted, there is no big qualitative change to the analysis, as the uncertain periods of life can be bundled together in order to let the results of the previous section to carry over to this case.

This extension reveals, however, a limitation to the possibility of compensating the loss of a short life. If life is uncertain right from the beginning, there is no possibility of compensating for the loss of a death at birth, and the perspective of concentrating consumption at the very beginning of life in order to compensate the loss of those who die in their very young years offers little hope for a good life for all.[21]

7 Savings and growth

Discouraging savings may be thought to have consequences on growth and capital accumulation. In the analysis up to now, time was ignored and the set of feasible allocations was not specified. The axiomatic analysis of Section 4 is in fact valid for any model with a finite number of agents, including a model with different cohorts. Moreover, Fleurbaey (2008) showed that this kind of axiomatic result can be extended to an infinite number of agents. We can therefore consider that the Maximin CCPERL is justified for a wide variety of models.

Now, the fact that the social criterion that is obtained puts absolute priority on the worst off has the well-known consequence, in a growth model, that the optimal path will typically involve stagnation, and this has nothing to do with the goal of compensating the short-lived. So, worrying about savings and growth should primarily be associated to the maximin before the compensation for premature death is considered.

In this section we propose to study a slightly different issue: What is the optimal steady-state pattern of consumption and savings in a standard growth model with overlapping generations, when the Maximin CCPERL is the criterion? This question

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[21]The 1969 movie "The Christmas Tree" shows the life of a young boy condemned to die from disease in his teens. His father tries to satisfy all the boy's wishes. Their relatives initially find this a very bad education but eventually realize that standard education is a long-term investment which makes no sense when life is short.

is relevant if one considers that there should be room for growth towards the optimal steady state when the economy is initially poor. In other words, the maximin may seem excessively egalitarian if it prevents growth toward the steady state, but it may appear quite relevant to characterize the optimal steady state.

Consider a discrete time, overlapping generations model in which population grows at the rate n. The individuals live either one or two periods, and to simplify matters we assume that preferences are uniform, described by the function $u(c) + \pi v(d)$, where π is the probability of surviving and also the proportion of survivors in every generation. Final utilities are $u(c)$ for the short-lived and $u(c) + v(d)$ for the long-lived. All individuals of a given generation are identical, except in longevity.

The technology exhibits constant returns to scale and the production per capita is equal to $f(k)$, where k is the quantity of capital per capita. The equation of motion of k is

$$(1 + n) k_{t+1} = (1 - \delta) k_t + f(k_t) - c_t - \frac{\pi}{1 + n} d_t,$$

where δ is the depreciation rate, c_t is the consumption of an agent born in t (and working in t), d_t is the consumption in period t of an agent born in $t - 1$.[22]

In a steady state, this equation becomes

$$c + \frac{\pi}{1 + n} d = f(k) - (\delta + n) k.$$

Assuming that the economy is rich enough, the optimal steady state for the CCPERL has $d = d^*$, for d^* defined by $v(d^*) = 0$, and c is maximized, which requires maximizing $f(k) - (\delta + n) k$. Therefore there is no difference about the optimal amount of capital from standard utilitarian theory. The golden rule $f'(k^*) = \delta + n$ still prevails.

It remains to see how the optimal steady state can be decentralized. One may wonder if discouraging savings is compatible with preserving the capital stock. The optimum life cycle plan for every individual is

$$c = f(k^*) - (\delta + n) k^* - \frac{\pi}{1 + n} d^*, \quad d = d^*.$$

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[22] With obvious notations, this per capita equation derives from the following equations in absolute quantities:

$$\begin{aligned} K_{t+1} &= (1 - \delta) K_t + N_t f(k_t) - N_t c_t - N_{t-1} \pi d_t \\ \frac{K_{t+1}}{N_{t+1}} &= \left(\frac{1 - \delta}{1 + n} \right) \frac{K_t}{N_t} + \frac{f(k_t)}{1 + n} - \frac{c_t}{1 + n} - \frac{\pi}{(1 + n)^2} d_t. \end{aligned}$$

Let us imagine per period budget constraints that include a tax on savings:

$$
\begin{aligned}
c + s &= f(k) - kf'(k) + \left(\frac{1-\pi}{1+n}\right) d_{-1} + \frac{\tau}{1+n} s_{-1}, \\
d &= (1 + r - \tau)s.
\end{aligned}
$$

In the first period, earnings are equal to $f(k) - kf'(k)$ and young agents also receive a redistribution of the savings left by the deceased, which is equal per capita to $\left(\frac{1-\pi}{1+n}\right) d_{-1}$.[23] We assume that this redistribution is equal among all young agents. In a model with dynasties one could imagine that the child of the deceased would inherit his parent's unused savings, and this would generate inequalities between dynasties. It simplifies the analysis to assume away such phenomenon. Finally, the savings tax is redistributed to the young as a lump-sum grant.

At the equilibrium one must have $r = f'(k) - \delta$ and

$$
\frac{u'(c)}{\pi v'(d)} = 1 + r - \tau.
$$

When $k = k^*$, and τ is adjusted so that the individuals choose $d = d^*$, the budget constraint implies that

$$
\begin{aligned}
c &= f(k) - kf'(k) + \left(\frac{1-\pi}{1+n}\right) d_{-1} + \frac{\tau}{1+n} s_{-1} - s \\
&= f(k^*) - (\delta + n) k^* - \frac{\pi}{1+n} d^*,
\end{aligned}
$$

which is the optimal amount of consumption.

In conclusion, the question of the life cycle profile of consumption seems relatively independent of the problem of accumulation in the standard growth model, at least in the steady state. Preserving the optimal stock of capital is possible even when a tax on savings induces the agents to plan a low consumption in their old age. Maximizing consumption in young age should be seen more as a transfer between generations (from old to young) who coexist *within* every period rather than as a transfer from the future to the present.

8 Short lives, big losses: evidence from France

We have argued so far that compensating individuals for a shorter life is feasible, despite uncertainty about individual longevities. But what would be the gain from

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[23]The subscript -1 means that this amount was fixed at the previous period and is exogenous when the agent makes his present decision.

implementing that compensation, in terms of welfare inequality reduction? To answer that question, we need to have an idea of the magnitudes, in the real world, of welfare inequalities due to unequal longevities. For that purpose, this section uses data on income and longevity in France (2008), and provides empirical estimates of the distribution of CCPERL among the population.

We consider that the adult life starts at age 20, and can potentially last until age 100.[24] We consider four socio-professional groups: executives, professionals, workers and employees. Individual preferences on lifetime consumption profiles are represented by a standard time-additive utility function:

$$\sum_{t=0}^{\lambda} \beta^t \left[\frac{c_{it}^{1-1/\gamma}}{1 - 1/\gamma} + \alpha \right]$$

where α, β and γ are preference parameters, λ is the length of adult life, while c_{it} is the consumption for an agent of socio-professional group i at period t (see the Appendix for the data). Parameters are calibrated as follows.[25]

Parameters	α	β	γ	T	ℓ^*
Values	-16.2	0.96	1.25	80	80

Table 1: Calibration of parameters

Let us first abstract from longevity inequalities, and focus on welfare inequalities due to income differentials. For that purpose, Figure 5 shows the CCPERL for the different socio-professional groups, under the assumption that each individual enjoys the same longevity, equal to average total lifetime in the population, i.e. 81 years (source: Blanpain 2011).[26] When longevity is set to its average level for all, the worst-off agents are, not surprisingly, the ones with the lowest income, i.e. employees. Their CCPERL is about 2000 euros less than that of the top group, i.e. executives. That gap is sizeable, but far smaller than the gap due to longevity inequalities, as suggested by Figure 6, which shows the CCPERL for different longevities, assuming that everyone enjoys the average consumption level. The CCPERL associated with the short life (35 years) is 5.5 times smaller than the CCPERL under a total life of 95 years. That huge gap gives an idea of the magnitudes of welfare losses due to premature deaths.

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[24]We will ignore childhood throughout this numerical example.

[25]Parameters α and γ are calibrated from Becker *et al* (2005). The time preference factor β coincides with a quarterly subjective discount factor of 0.99 (see de la Croix and Michel 2002).

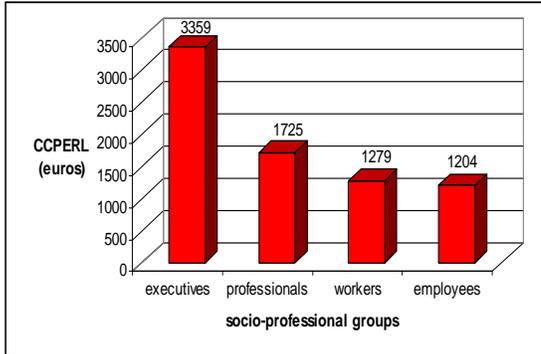[26]Calculations are left to the Appendix.
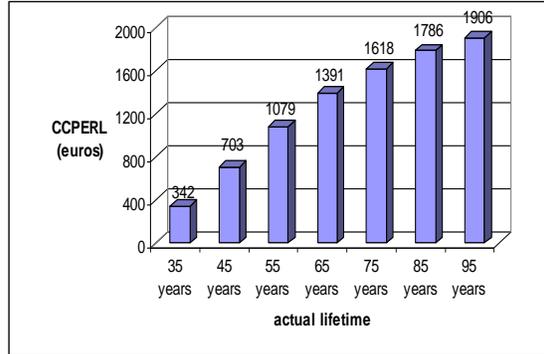
Figure 5: CCPERL under average
lifetime

Figure 6: CCPERL under average
income

When computing the distribution of CCPERL for different longevities and socio-professional groups (Figure 7), we see that the welfare gap due to a different socio-professional group is increasing with the length of life, but at a declining rate. The CCPERL gradient seems to be steeper along the longevity dimension than along the socio-professional dimension when considering *short* lives, but steeper along the socio-professional dimension when we consider *long* lives. This suggests that, even if little can be done for individuals dying on the age-interval $[35, 55]$, more can be done, thanks to income redistribution, for those dying on the age-interval $[55, 75]$.
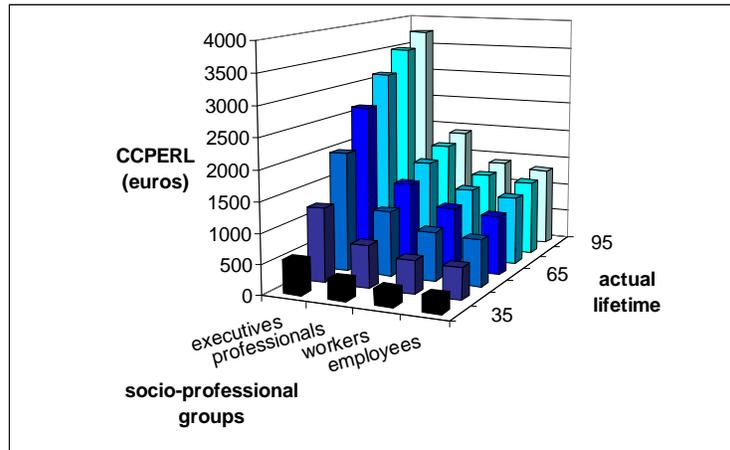

Figure 7: distribution of CCPERL by
socio-professional groups and lifetimes

The CCPERL of executives dying at age 55 exceeds the CCPERL of individuals in other socio-professional groups living much longer. Hence, income redistribution from executives older than 55 years towards younger individuals from other groups would constitute a major step towards the compensation of those dying in the $[55, 75]$ age-interval. Such a redistribution would also reduce the prevalence of poverty.[27]

Finally, let us compares the welfare losses due to a premature death with the ones due to a lower consumption. For that purpose, Table 2 contrasts the CCPERL gain (resp. loss) due to a longer (resp. shorter) life than the average longevity, with the CCPERL gain (resp. loss) due to a consumption larger (resp. smaller) than the average consumption. The latter differential appears in brackets.

Longevity	executives	professionals	workers	employees
35 years	-2811	-1383	-1000	-936
	(206)	(0)	(-63)	(-74)
45 years	-2124	-1022	-731	-683
	(532)	(0)	(-155)	(-182)
55 years	-1367	-646	-458	-427
	(913)	(0)	(-258)	(-302)
65 years	-717	-334	-235	-219
	(1251)	(0)	(-347)	(-406)
75 years	-231	-107	-75	-70
	(1510)	(0)	(-414)	(-484)
85 years	132	61	43	40
	(1705)	(0)	(-464)	(-542)
95 years	393	181	127	118
	(1846)	(0)	(-500)	(-584)

Table 2: CCPERL gaps due to deviations in longevity or consumption

CCPERL losses and gains arising from deviations around average longevity are large, in comparison to those coming from deviations in consumption. For instance, raising the longevity of an employee dying at age 35 to the average longevity would increase his CCPERL by 936 euros, whereas raising his consumption to the average level would raise his CCPERL by only 74 euros. Moreover, CCPERL losses (resp. gains) arising from living less (resp. more) than the average are increasing with the

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[27]If one fixes the poverty threshold to a CCPERL equal to 485 euros (see the Appendix), the poor include all employees, workers and professionals dying before reaching the age of 45. Redistributing towards them in such a way that they enjoy the CCPERL of executives with the *same* longevity (equal to 548 euros) would eradicate poverty in our sample.

consumption level.[28] Note also that CCPERL gains and losses related to consumption differentials, although of smaller magnitudes, are nonetheless significant, and are strongly increasing in longevity.[29]

Short lives, big losses. Although it has no pretension to exhaustiveness, our estimation of the distribution of CCPERL for France illustrates that premature deaths are at the root of substantial welfare inequalities. Undoubtedly, such magnitudes reinforce the case for compensation.

9   Concluding remarks

Can one compensate the dead? Such a compensation seems impossible, as short-lived persons are hard to identify *ex ante*, and, once dead, it is too late. However, this study provides a positive answer: one solution is to allocate resources *ex ante* in such a way as to maximize the minimum Constant Consumption Profile Equivalent on the Reference Lifetime. The Maximin CCPERL solution involves, in general, declining consumption profiles, and provides a substantial, and sometimes full, compensation to the short-lived.

While concern for social inequalities in life expectancy is commonplace, the possibility to implement compensation for premature deaths by encouraging consumption earlier in life is seldom discussed in policy debates. However, the familiar concern with poverty in old age can be seen as the opposite side of the coin. When people do not or cannot save enough for their old age, living long becomes a burden and guaranteeing a decent minimum can be seen as an insurance against the bad luck of a long life with insufficient resources.

With growing affluence in rich countries, the problem of poverty in old age recedes and the injustice of premature deaths becomes more serious the more affluent the elderly people are. Fighting the causes of premature deaths is a very popular cause. What we suggest here is that, in addition, encouraging early consumption (or similar goods such as leisure) could also help reducing the injustice. The tradition of encouraging savings and fighting myopia, which comes from an era of scarcity but remains strong nowadays, appears somewhat ill-adapted to a context of affluence.

There are various aspects of the problem that have been neglected in this paper and are left for future work. First, while giving absolute priority to the worst off comes out neatly from the axiomatic analysis and simplifies the analysis of policies

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[28]Executives dying at age 95 enjoy a CCPERL gain of 393 euros in comparison to those enjoying average longevity, whereas, for employees, the CCPERL gain from such a long life equals 118 euros.

[29]For instance, raising the consumption of an employee would increase his CCPERL by only 74 euros in case of death at age 35, while it would raise it by 584 euros under death at age 95.

because it makes it sufficient to identify the worst off under each policy, it may have extreme consequences in some contexts and one might want to study less extreme criteria. We have seen for instance that putting some long-lived people below subsistence could be the outcome of the maximin criterion when this gives a benefit to the worst off. This kind of policy would be less likely under a criterion with a more moderate degree of priority to the worse off.

Second, as mentioned in the introduction, some longevity inequalities result from individual behaviour, not simply from natural factors. One might think that, intuitively, risk-taking short-lived agents should be less compensated than other short-lived persons, as they bear some responsibility for their short life. Incorporating this element of responsibility in the analysis of compensation for a short life requires a substantial extension of the setting and a reexamination of the social objective. This is the object of a companion paper by the same authors.

Third, the analysis of policy in a growth model has remained very simple in the previous section and it would be interesting to study a richer model containing endogenous fertility and mortality, with health expenditures that reduce mortality competing with ordinary consumption. This is studied in Ponthiere (2011). Inequalities between dynasties due to the random distribution of premature deaths is also an interesting topic to be studied. More generally, the role of bequests should be analyzed explicitly, as for instance the warm glow of leaving savings to one's offspring may partly reduce the loss of a premature death.

Fourth, we have not analysed the problem that occurs when wealth inequalities are important and require to combine redistribution of income and incentives on savings. This would need a richer model in which incentive constraints limit the scope of income redistribution. In such a model, it may happen that the less wealthy agents suffer from poverty in old age while the wealthier agents have the opposite problem. We have observed that inducing old age consumption to be at the neutral level d^* for all may eliminate inequalities of any direction between short-lived and long-lived, but a more thorough analysis of the interaction between the income tax and the savings tax would be interesting.

Fifth, the political economy of compensation policies would deserve a separate study. At first glance, one may think that the Maximin CCPERL policy should find little support in the population. The young would not welcome a constraint on their saving, and the elderly people would not welcome the prospect of a lower pension. This, however, assumes that people only vote for their immediate interest, which contradicts the observation that the most generous often come from the upper middle class. If people are concerned with social welfare, they may realize that the worse off are those who suffer from premature death and they may therefore accept

to endure a general boost on early consumption for the sake of the unlucky.

In conclusion, the main point of our paper is to offer a new perspective on the problem of the allocation of consumption over the life cycle. Traditional perspectives have emphasized the role of intertemporal preferences for consumption smoothing, of myopia and hyperbolic discounting, and of imperfect financial markets which contribute to generating the observed inverted-U shape in consumption profiles (Lee and Tuljapurkar 1997). The additional consideration we propose to add is that the greater the consumption in old age, the greater the injustice endured by those who die prematurely. While we make the point that policy intervention can do something about it, we leave it open how to incorporate this perspective into a complete outlook of life-cycle management.

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11 Appendix

11.1 Proof of Theorem 1

Lemma 1 *Assume that the social ordering function $\succsim$ satisfies Axioms 1-2-3 on $\Re^{|N|}$. Then $\succsim$ is such that for all $R_N \in \Re^{|N|}$, all $x_N, y_N \in X^{|N|}$, and all $i, j \in N$, if $R_i = R_j$ and*

$$y_i P_i x_i P_i x_j P_j y_j$$

and $x_k P_k y_k$ for all $k \neq i, j$, then

$$x_N \succsim_{R_N} y_N.$$

Proof. If $\lambda(x_i) = \lambda(y_i) = \lambda(x_j) = \lambda(y_j)$, the result follows from Fleurbaey and Maniquet (2011, Lemma A.1).

By assumption on $\Re$, there exist $\bar{x}_N, \bar{y}_N$ such that:

1) $\lambda(\bar{x}_i) = \lambda(\bar{y}_i) = \lambda(\bar{x}_j) = \lambda(\bar{y}_j) = T$;

2) $\bar{y}_i P_i' y_i P_i' x_i P_i' \bar{x}_i P_i' x_j P_j' \bar{x}_j P_j' \bar{y}_j P_j' y_j$;

3) $x_k P_k \bar{x}_k P_k \bar{y}_k P_k y_k$.

By Weak Pareto, $x_N \succ_{R_N} \bar{x}_N$ et $\bar{y}_N \succ_{R_N} y_N$. By the previous step, $\bar{x}_N \succsim_{R_N} \bar{y}_N$. By transitivity, $x_N \succsim_{R_N} y_N$. ∎

Lemma 2 *Assume that the social ordering function $\succsim$ satisfies Axioms 1-2-3-4 on $\Re^{|N|}$. Then $\succsim$ is such that for all $R_N \in \Re^{|N|}$, all x_N, $y_N \in X^{|N|}$, and all i, $j \in N$, if*

$$\hat{y}_i > \hat{x}_i > \hat{x}_j > \hat{y}_j$$

and $x_k P_k y_k$ for all $k \neq i, j$, then

$$x_N \succsim_{R_N} y_N.$$

Proof. The allocations constructed in this proof all involve a longevity equal to ℓ^* and a constant consumption for i, j (this will not be repeated below). Let R^* denote the Leontief preferences represented by $\min \{ x_{ik} \mid x_{ik} > 0, \ 1 \leq k \leq T \}$.

Let z_N^1, z_N^2, z_N^3 be such that:
1) $z_i^1 > \hat{y}_i > z_i^2 > \hat{x}_i > z_i^3 > \hat{x}_j > z_j^3 > z_j^2 > z_j^1 > \hat{y}_j$;
2) $z_i^2 - z_i^3 = z_j^3 - z_j^2$;
3) $x_k P_k z_k^3 = z_k^2 P_k z_k^1 P_k y_k$ for all $k \neq i, j$.
Let R_N', R_N'' be such that:
1) $I(x_k, R_k') = I(x_k, R_k)$, $I(y_k, R_k') = I(y_k, R_k)$ for $k = i, j$;
2) $I(x_k, R_k'') = I(x_k, R_k)$, $I(z_k^1, R_k'') = I(z_k^1, R_k)$ for $k = i, j$;
3) $I(z_i^1, R_i') = I(z_i^1, R^*)$, $I(z_i^2, R_i'') = I(z_i^2, R^*)$, and $R_i'' = R_j''$;
4) $R_k' = R_k'' = R_k$ for all $k \neq i, j$.
Suppose that, contrary to the desired result, one has $y_N \succ_{R_N} x_N$. By Axiom 2, $y_N \succ_{R_N'} x_N$. By Axiom 1, $z_N^1 \succ_{R_N'} y_N$ and by transitivity, $z_N^1 \succ_{R_N'} x_N$. By Axiom 2, $z_N^1 \succ_{R_N''} x_N$. By Lemma 1, $z_N^2 \succsim_{R_N''} z_N^1$ and by transitivity, $z_N^2 \succ_{R_N''} x_N$. By Axiom 4, $z_N^3 \succsim_{R_N''} z_N^2$ and by transitivity, $z_N^3 \succ_{R_N''} x_N$. The latter contradicts Axiom 1. Therefore $x_N \succsim_{R_N} y_N$. ∎

The rest of the proof of Prop. 1 is a standard argument (see Hammond 1979 or Fleurbaey and Maniquet 2011).

11.2 Short lives, big losses: data and calculations

Consumptions Consumption levels are approximated by labour incomes during the working period (until age 60), and by pension benefits during retirement (starting at age 60). Table 3 shows the raw data on monthly wage and pension benefits in France (2008), from which annualized consumptions are computed.[30]

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[30]Sources: INSEE (2011). The amounts represent average monthly income (net of taxes), in euros. To compute the retirement pension benefit, we applied a uniform 70% replacement rate on monthly income, which coincides with the average replacement rate in France. Changing this would not affect our results significantly.

	executives	professional	workers	employees	total
monthly wage	4081	2068	1523	1432	2068
monthly pension benefit	2857	1448	1066	1002	1448

Table 3: Monthly incomes and pension benefits in France (in euros)

Figure 5 The CCPERL for group i is the solution x to the equation:

$$\sum_{t=0}^{60}(0.96)^t\left[\frac{c_{it}^{1-1/(1.25)}}{1-1/(1.25)}-16.2\right]=\sum_{t=0}^{79}(0.96)^t\left[\frac{x^{1-1/(1.25)}}{1-1/(1.25)}-16.2\right]$$

Figure 6 The CCPERL under an adult life of length λ is the solution x to:

$$\sum_{t=0}^{\lambda}(0.96)^t\left[\frac{c_t^{1-1/(1.25)}}{1-1/(1.25)}-16.2\right]=\sum_{t=0}^{79}(0.96)^t\left[\frac{x^{1-1/(1.25)}}{1-1/(1.25)}-16.2\right]$$

where c_t is the average consumption at period t. Average monthly consumption when being active is set to 2068 euros, against 1448 when being retired.

Figure 7 The CCPERL under an adult life of length λ for a group i is the solution x to:

$$\sum_{t=0}^{\lambda}(0.96)^t\left[\frac{c_{it}^{1-1/(1.25)}}{1-1/(1.25)}-16.2\right]=\sum_{t=0}^{79}(0.96)^t\left[\frac{x^{1-1/(1.25)}}{1-1/(1.25)}-16.2\right]$$

Poverty measurement The poverty threshold is computed relying on the amounts of minimum benefits provided by the French State. It consists in giving a minimum allowance of 448 euros and a minimum pension benefit of 709 euros. The poverty threshold CCPERL is then a solution to:

$$\sum_{t=0}^{79}(0.96)^t\left[\frac{\tilde{c}_t^{1-1/(1.25)}}{1-1/(1.25)}-16.2\right]=\sum_{t=0}^{79}(0.96)^t\left[\frac{x^{1-1/(1.25)}}{1-1/(1.25)}-16.2\right]$$

where $\tilde{c}_t$ is the consumption at the poverty level. The CCPERL poverty threshold presupposes that longevity equals $\ell^* = 80$. If we had taken average longevity, that CCPERL threshold would have been 429 euros.